MESSAGE

14 Balance Is Basic

A 21ST-CENTURY VIEW OF A BALANCED MATHEMATICS PROGRAM

Even though much has changed about the nature of mathematics students need and the depth of mathematical thinking called for today, the fundamentals of a balanced mathematics program remain much the same as they have for years. Essentially, we want all students to

1. *make sense of math:* understand mathematical concepts and ideas so they can make sense of the mathematics they do;
2. *do math:* know mathematical facts and perform mathematical skills; and
3. *use math:* solve a wide range of problems in various contexts by reasoning, thinking, and applying the mathematics they have learned.

However, even though we may agree on the broad categories, arguments abound over what exactly each of these pieces comprises and over the relative weight and importance of each. People are often quick to categorize opposing views, applying labels like *back to basics* or *reform*. For example, some people fear that students in a program they label as *reform* might not receive the emphasis on computation they see in more traditional programs. Others fear that if students focus excessively on computational procedures without adequate understanding, their proficiency will be superficial and short-lived. Still others argue that if students cannot solve particular types of problems, ranging from routine word problems to complex problems with no known solution, they will not be prepared for the world outside of school. It's time to get past these arguments, move beyond the labels, and look at what we mean when we discuss the components of a balanced mathematics program.

Making Sense of Math: Conceptual Understanding

Without conceptual understanding, learning skills is meaningless. Many students acquire skills (with varying degrees of success), only to find that they lack the understanding that would help them apply what they have learned. For example, in solving a problem involving fractions, a student may know the rules: how to *go straight across* (when multiplying fractions), *turn upside down and then go straight across* (when dividing fractions), or *find a common denominator* (for adding and subtracting fractions with unlike denominators). But what if a student doesn't understand the operations the rules represent or how they relate to the problem at hand? In such a case, every encounter with fractions results in a guessing game: Which strategy should I use? Even students skilled in procedures will be lost if they don't understand the meanings of the operations well enough to determine which procedure might be appropriate.

On the other hand, if a student has a strong understanding of what a fraction represents, and also a solid understanding of what it means to add, subtract, multiply, or divide, the act of adding fractions becomes far less complex—and less of a guessing game. Students with both skills and conceptual understanding are likely to more easily form mental images of mathematical concepts they might recognize in problem situations. As a result, a student can see in her mind how to put the fractions together for the situation a problem presents. She has no reason to be tempted to choose an incorrect rule because she knows what operation(s) might be helpful and which procedure(s) could therefore be useful.

Doing Math: Skills, Facts, and Procedures

Understanding mathematical ideas and concepts is powerful. But without also knowing how to perform the necessary skills, understanding alone is insufficient. If taught well, proficiency with mathematical skills can contribute to mathematical understanding.

Both components—skills and understanding—are especially critical when tackling challenging problems.

In today's highly technological world, there is debate as to the extent of computational skills a student needs. We all want students to know how to add, subtract, multiply, and divide whole numbers, decimals, and fractions for both positive and negative numbers. But we must also be realistic about when students should be able to compute mentally, when they should perform computation with a pencil, and when they might rely on a calculator or other technological tool. It may be that being able to use a pencil and paper to do a long-division exercise with a four-digit divisor is not as critical as it once was. Perhaps students have as much proficiency with the tool of division as they need if they can perform pencil-and-paper division exercises through two-digit divisors, for example, and longer or more complex division exercises with a calculator. Students might be expected to estimate results of most division exercises mentally, and even determine exact answers to certain types of basic division exercises in their heads. These shifts don't make computational skills unimportant. Rather, the shifts help us make room for the rest of a balanced program that includes the solid understanding described earlier and what is arguably the most critical feature of a balanced mathematics program—problem solving.

Using Math: Problem Solving

If students have a sound understanding of mathematical ideas and some level of skills mastery, they have certainly gained important mathematical knowledge. But there is a difference in knowledge that is useful for an exercise in a mathematics textbook and knowledge that is needed to solve authentic, multifaceted problems in other contexts. For a student's knowledge of mathematics to truly come together, we must also focus on the third component of a balanced mathematics program: problem solving. Solving problems requires that students consider the many tools they have acquired and select which one(s) might be useful for a wide range of problems at varying levels of complexity and in many different contexts. Solving problems is the most visible indicator of how well a student has assimilated the knowledge he has acquired and how comprehensive a set of mathematical tools the student has built.

Beyond the Big Three

What constitutes a balanced mathematics program has been the subject of significant work in mathematics education for more than thirty years. The National Council of Supervisors of Mathematics released a statement about *new basics* in 1977, supporting the teaching of an expanded set of basic skills that calls for (among other things) a focus on problem solving and the incorporation of appropriate uses of technology. Shortly after, in 1980, the National Council of Teachers of Mathematics (NCTM) produced its call to action entitled *An Agenda for Action*, which outlines priorities for school mathematics. These priorities center on problem solving and call for the use of tools like calculators and computers. NCTM took an even bolder step at the end of the same decade in launching their landmark publication, *Curriculum and Evaluation Standards for School Mathematics* in 1989, in which a comprehensive mathematics program for grades K–12 is outlined, elaborating what mathematical content and processes all students need. In 2000, NCTM further refined the agenda with the publication of *Principles and Standards for School Mathematics*. All of these publications reinforce the importance of a balanced program of conceptual understanding, skills, and problem solving, while also reflecting shifts in emphasis within each of these. These publications also recognize the growing importance of what we might consider the connective tissue of a coherent mathematics program—mathematical habits of mind, flexible thinking skills, and strong quantitative reasoning. These critical abilities mirror the twenty-first-century skills now called for in report after report from the business and policy sectors. In continuing to expand our thinking about priorities we now realize the importance of helping students show or represent mathematical situations in different ways (using words, objects, pictures, graphs, tables, numbers, and symbols). As students move fluently among these, they solidify their understanding and, potentially, their proficiency. We also now recognize how important it is for students to communicate the mathematics they know. Presenting or explaining an idea or a solution to a problem can help a student reinforce what he knows or, alternatively, can help him realize (and correct) errors in his thinking.

In *Adding It Up* (Kilpatrick, Swafford, and Findell 2001) and its partner booklet, *Helping Children Learn Mathematics* (Findell and Swafford 2002), the authors use the metaphor of a rope to define mathematical knowledge. The five strands of the rope represent understanding, computing, applying, reasoning, and engaging. The first three of these strands reflect the three components of balanced mathematics discussed in this message. The last two—reasoning (and the various types of mathematical thinking associated with it) and engaging (including habits of mind such as persistence and a willingness to take on a challenging problem)—reflect the connective tissue previously described.

What Can We Do?

Whether you prefer the metaphor of a rope or the notion of three building blocks with connective tissue, the fact is that all students need it all. They need a balanced program of understanding, skills, and problem solving and they need a flexible set of thinking and reasoning tools they can call on to pull all of these pieces together. From a mathematical standpoint, each piece needs to support the other pieces. Students need to connect understanding with doing and using mathematics. They need to use tools of communication, representation, reasoning, and thinking to make mathematics useful beyond the classroom. Whenever we omit part of a balanced mathematics program for any student, whatever is left falls apart. Whether intentional or unintentional, whether guided by good intentions or low expectations, whether targeted at one student or at a group of students, the student without a balanced and comprehensive knowledge of mathematics has no foundation upon which to build future mathematical success.

It only makes sense, from a mathematical perspective and from a moral and ethical perspective, for schools to absolutely commit to providing all students a deep, connected, comprehensive, balanced mathematics program that will allow every student to meet the increasing demands of twenty-first-century society and the workplace.

Reflection and Discussion

FOR TEACHERS

- What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
- How can you connect the skills students learn with their understanding of what the skills represent?
- How can you help students develop mathematical habits of mind such as the willingness to take on challenging tasks or the perseverance to spend time on a hard problem?
- Which components of a balanced mathematics program do you feel you are most effectively providing for your students? Which components might you more effectively target in the future?

FOR FAMILIES

- What questions or issues does this message raise for you to discuss with your son or daughter, the teacher, or school leaders?
- How can you work together with your daughter or son to communicate how important it is to understand or make sense of the procedures she or he is learning?
- What messages can you convey to your son or daughter about the importance of connecting the pieces of mathematics being learned? Make a point to ask your son or daughter to explain to you a skill he or she is learning and how it connects or relates to the learning of the previous week.
- In what ways might you ask your daughter or son to show you another way to think about a problem or situation she or he is working on? Consider the use of pictures, models, a table, and so on.

FOR LEADERS AND POLICY MAKERS

- How does this message reinforce or challenge policies and decisions you have made or are considering?
- How balanced is your mathematics curriculum, and how well do your textbooks or instructional materials reflect a comprehensive and connected blend of understanding, skills, and problem solving? How well does your program incorporate the critical connective tissue that includes reasoning and mathematical thinking? Addressing these issues in the curriculum will likely call for careful study by knowledgeable mathematics teachers or other mathematics educators.
- How can you help your teachers teach in ways that help students develop mathematical habits of mind?